LLNL-JRNL-403826

LAWRENCE LIVERMORE NATIONAL LABORATORY

# Aperture Effects and Mismatch Oscillations in an Intense Electron Beam

J. R. Harris, P. G. O'Shea

May 13, 2008

Physics of Plasmas

# Aperture Effects and Mismatch Oscillations in an Intense Electron Beam

John R. Harris

Lawrence Livermore National Laboratory, Livermore, CA 94551

Patrick G. O'Shea

Department of Electrical and Computer Engineering, University of Maryland, College Park, MD 20742

## ABSTRACT

When an electron beam is apertured, the transmitted beam current is the product of the incident beam current density and the aperture area. Space charge forces generally cause an increase in incident beam current to result in an increase in incident beam spot size. Under certain circumstances, the spot size will increase faster than the current, resulting in a decrease in current extracted from the aperture. When using a gridded electron gun, this can give rise to negative transconductance. In this paper, we explore this effect in the case of an intense beam propagating in a uniform focusing channel. We show that proper placement of the aperture can decouple the current extracted from the aperture from fluctuations in the source current, and that apertures can serve to alter longitudinal space charge wave propagation by changing the relative contribution of velocity and current modulation present in the beam.

## 1. Introduction

Apertures, the simplest means for controlling electron beams, have been used since the earliest research on cathode rays [1]. They remain in use today to alter beam current and intensity [2], to affect beam transverse and longitudinal profiles [3-5], for transverse focusing [6], and in beam diagnostics [7,8]. At the most fundamental level, an aperture produces an output beam whose current is proportional to the current density of the incident beam, provided that the incident beam size is larger than the aperture. As the incident beam current is changed, the size of the beam striking the aperture will generally also change. Under certain conditions, space charge can cause the beam cross-sectional area to increase faster than the beam current, so that the incident current density and the extracted current both decrease. For a gridded electron gun, this will mean that as the grid voltage is made more positive with respect to the cathode, the current extracted from the aperture will decrease, rather than increase as expected. Such a system can be viewed as exhibiting a negative value of the transconductance

$$g_{m,A} = \frac{\partial I_3}{\partial V_{GK}}, \tag{1}$$

where $I_3$ is the current extracted from the aperture, $V_{GK}$ is the grid-cathode voltage, and the subscript $A$ denotes the transconductance of the aperture-gun system. By analogy with triodes, the transconductance $g_m$ of an unapertured electron gun is positive, and is proportional to the cube-root of the beam current [9]. For the general axisymmetric transport system shown in Figure 1, the transconductance $g_{m,A}$ of the aperture-gun system is related to the transconductance $g_m$ of the unapertured gun by

$$g_{m,A} = g_m \frac{r_3^2}{r_2^2}\left[1 - 2\frac{I_1}{r_2}\frac{\partial r_2}{\partial I_1}\right], \tag{2}$$

where $r_2(I_1)$ is the beam radius incident on the aperture, $r_3$ is the aperture radius, and $I_1(V_{GK})$ is the current extracted from the gun [10]. The ratio $g_{m,A}/g_m = \partial I_3/\partial I_1$ denotes the change in the transmitted current resulting from a change in the incident current, and retains this interpretation even when the beam is not extracted from a gridded gun. The function $r_2(I_1)$ is determined by the details of the transport system between the gun and aperture. We previously considered the case of an intense beam expanding radially under space charge forces in the absence of transverse focusing, and showed that when apertured it can exhibit negative transconductance [10]. In this paper, we will apply these concepts to an intense, axisymmetric electron beam propagating in a uniform focusing channel, and consider the effects of the apertures on space charge waves propagating in the beam.

## 2. Uniform Focusing, Mismatch, and Apertures.

### A. Envelope Oscillations.

We begin with some preparatory comments on matching and intense beams before considering the beam behavior at several locations along a focusing channel. Consider an intense beam of current $I_1$ injected on axis into a uniform axial magnetic field with transverse focusing strength [11] $k_0 = qB/2mc\gamma\beta$, where $q$ is the electron charge, $B$ is the magnetic induction, $m$ is the electron mass, $c$ is the speed of light, and $\beta$ and $\gamma$ are the relativistic factors. In the limit of negligible emittance, this beam will be matched if it is injected with zero divergence ($dr_1/ds = 0$) and with a radius $r_1 = a_B$, where

$$a_B = \frac{\sqrt{K}}{k_0} \tag{4}$$

is the Brillouin radius,

$$K = \frac{2I_1}{I_0 \beta^3 \gamma^3} \tag{5}$$

is the generalized perveance, and $I_0$ is the characteristic current (17 kA for electrons) [11]. This matched beam will continue to propagate through the focusing channel without change in radius. For a given initial radius, beam energy, and focusing channel strength, there is only one choice of beam current that will result in a matched beam; call this value the matched current $I_M$. If the current is now changed from $I_M$ to a new value $I$, the beam will undergo mismatch oscillations about the new equilibrium radius $a_B(I)$ calculated from Eqs. (4) and (5) but using the new current $I$. If the amplitude of the mismatch oscillation is small[1], the oscillation is simple harmonic motion, described by a sine function. In this limit, the envelope mismatch wavelength is [11]

$$\lambda_e = \frac{2\pi}{\sqrt{2}k_0 \left[ 2 - \frac{K}{k_0^2 {a_B}^2} \right]^{1/2}} . \tag{6}$$

Here, $a_B$ is the new equilibrium radius. The quantity $K / k_0^2 {a_B}^2$ is known as the intensity parameter, and varies from 0 in the limit of negligible space charge to 1 in the limit of neglible emittance [2]. So the envelope oscillation wavelength for our intense beam becomes

$$\lambda_e = \frac{2\pi}{\sqrt{2}k_0} = 2\sqrt{2}\pi \frac{mc\gamma\beta}{Bq} . \tag{7}$$

[1] For this discussion, we take "small" to mean a mismatch oscillation of less than 10% of the beam radius, which limits changes in the beam current to less than 20%.

This wavelength depends only on the magnetic field and beam energy, and not on the beam current.

B. Beam at $s = 0$.

First, consider a beam with initial radius $r_1$, which is matched for a current $I_M$. If the current is increased or decreased, the beam will undergo mismatch oscillations. However, the radius at injection, $r_1$, is an initial condition which is independent of the beam current. The beam radius at $s = 0$ will always be $r_1$, and due to the periodic nature of the mismatch oscillation, the beam radius in the focusing channel ($r_2$) will return to this value after traveling integer multiples of the mismatch wavelength (Fig. 2). Referring to Eq. (2), this means that $\partial r_2 / \partial I_1 = 0$ at these locations, so that if an aperture of radius $r_3$ is placed there, the system transconductance becomes

$$g_{m,A} = g_m \frac{r_3^2}{r_1^2}. \tag{8}$$

The system transconductance at these locations does not depend on beam current, will always be positive, and will always be less than that of the unapertured beam.

C. Beam at $s = \frac{1}{2}\lambda_e$.

Now, consider the beam at one-half envelope wavelength downstream from the point of injection. This is the location of the maximum beam radius if the beam current exceeds the matched current, and the location of the beam waist if the current is less than the matched current. In either case, the beam radius at $s = \frac{1}{2}\lambda_e$ will be the sum of the equilibrium radius ($a_B(I_1)$), and the difference between the equilibrium and injection radii ($a_B(I_1) - r_1$). So

$$r_2 = 2a_B(I_1) - r_1, \tag{9}$$

which depends on the actual beam current through the Brillouin radius. The beam will repeat this condition once every wavelength at $s = \frac{1}{2}\lambda_e, \frac{3}{2}\lambda_e, \frac{5}{2}\lambda_e....$ If the beam is apertured at these locations, Eqs. (2), (4), (5), and (9) can be used to find the system transconductance

$$g_{m,A} = -g_m \frac{r_3^2}{r_1^2}\left[\frac{2a_B(I_1)}{r_1} - 1\right]^{-3}.$$

(10) Since $r_1$ is the Brillioun radius for the matched current $I_M$, Eqs. (4), (5), and (10) give

$$g_{m,A} = -g_m \frac{r_3^2}{r_1^2}\left[2\sqrt{\frac{I_1}{I_M}} - 1\right]^{-3}. \tag{11}$$

This transconductance will be negative provided $I \geq I_M/4$; in the limit of small changes in beam current $I_1 \to I_M$, so that

$$g_{m,A} \to -g_m \frac{r_3^2}{r_1^2}. \tag{12}$$

D. Beam at $s = \frac{1}{4}\lambda_e$

Twice in each envelope oscillation period (at $s = \frac{1}{4}\lambda_e, \frac{3}{4}\lambda_e, \frac{5}{4}\lambda_e...$), the local beam radius will be exactly equal to the Brillouin radius, $r_2 = a_B$. Since the Brillouin radius depends on the square root of the beam current, the current density at these locations will always stay the same -- the change in beam current is exactly counteracted by the change in beam cross-sectional area. This gives a transconductance of the apertured beam of

$$g_{m,A} = 0\,. \tag{13}$$

This is an interesting result, because it means than an aperture placed at these locations will serve to isolate everything downstream of the aperture from beam current variations produced in the source. Like all results discussed in this paper, this is true regardless of whether the beam is produced from a gridded gun or by some other means. In the latter case, the source transconductance $g_m = \partial I_1 / \partial V_{GK}$ can be replaced by the derivative of beam current with respect to some other control variable, such as laser intensity for a photocathode.

E. Limitations.

Equation (2) is only valid if the incident beam radius is larger than the aperture radius, $r_2 > r_3$. If this does not hold, the entire beam passes through the aperture and $g_{m,A} = g_m$. For $s = 0$, and similar locations, this requirement is equivalent to

$$\frac{g_{m,A}}{g_m} < 1\,. \tag{14}$$

For $s = \lambda_e$, and similar locations, this requirement becomes $r_3 < 2a_B - r_1$, or (using Eq. (2))

$$\frac{g_{m,A}}{g_m} > \frac{1}{1 - 2\sqrt{\dfrac{I}{I_M}}}\,. \tag{15}$$

These limiting curves are plotted in Fig. 3, along with the $g_{m,A} / g_m$ curves for several values of $r_3 / r_1$.

## 3. Application to Space Charge Waves

Consider a beam arriving at the aperture with current

$$I_1(t) = I_{1,0} + i_1(t) \tag{16}$$

and velocity

$$V_1(t) = c\beta + v_1(t), \tag{17}$$

where the current modulation

$$i_1(t) = \eta_1 I_{1,0} h(t) \tag{18}$$

and the velocity modulation

$$v_1(t) = \delta_1 c\beta h(t) \tag{19}$$

are small perturbations compared to the unmodulated current $I_{1,0}$ and velocity $c\beta$, $\eta_1$ and $\delta_1$ define the strengths of the current and velocity modulation, and $h(t)$ is a dimensionless function varying between 1 and 0 which defines the shape of the modulation. Such modulations will launch fast and slow space charge waves. The behavior of these waves is governed by the ratio $\eta / \delta$ [12], which for a gridded electron gun depends on the anode, cathode, and grid voltages, the gun's mode of operation, and its amplification factor [13]. The function $h(t)$ also plays a role in determining the nature of conversion between the kinetic energy and potential energy associated with the waves as they evolve [14].

Small-amplitude current modulation of the type discussed here will cause portions of the beam to have more or less than the matched current, and therefore to undergo mismatch oscillations. If the current in each slice of the beam is conserved, each slice can be treated independently and assumed to have a mismatch oscillation amplitude based solely on the current in that slice, so that the results of the previous section can be applied to the modulated beam. Space charge wave propagation will change the current in each slice of the beam, but such longitudinal evolution of the beam occurs much more slowly than changes in the beam envelope. Therefore, we now assume that the beam arrives at an aperture which is sufficiently close to the gun so that no appreciable evolution of the modulation has occurred due to propagation of the space charge waves. Eq. (2) defines the ratio of the apertured transconductance to the transconductance of the unapertured gun, which can be rewritten

$$\frac{g_{m,A}}{g_m} = \frac{\partial I_3 / \partial V_{GK}}{\partial I_1 / \partial V_{GK}} = \frac{\partial i_3 / \partial t}{\partial i_1 / \partial t}. \tag{20}$$

Therefore,

$$\frac{\partial i_3}{\partial t} = \frac{\partial i_1}{\partial t}\left(\frac{g_{m,A}}{g_m}\right) \tag{21}$$

or

$$\frac{\partial i_3}{\partial t} = \eta_1 I_{1,0} \frac{\partial h(t)}{\partial t}\left(\frac{g_{m,A}}{g_m}\right). \tag{22}$$

If the modulation of the beam emerging from the aperture is similarly given by

$$i_3(t) = \eta_3 I_{3,0} h(t), \tag{23}$$

then

$$\eta_3 = \eta_1 \frac{I_{1,0}}{I_{3,0}}\left(\frac{g_{m,A}}{g_m}\right). \tag{24}$$

The current ratio in this equation is just the ratio of the steady-state beam area incident on the aperture to the area of the aperture itself, so this equation becomes

$$\eta_3 = \eta_1 \left( \frac{R_2}{r_3} \right)^2 \left( \frac{g_{m,A}}{g_m} \right), \tag{25}$$

in which the incident beam radius is denoted by a capital letter to indicate that it is the steady-state value corresponding to the unmodulated beam. Since the aperture has no effect on the velocity perturbation in the beam, the role of the aperture is to change the value of $\eta / \delta$, and therefore to change the nature of the space charge wave evolution downstream of the aperture.

For an intense beam propagating in a uniform focusing channel, as discussed in the previous sections, the change in $\eta / \delta$, and therefore the downstream behavior of the space charge waves, will depend on the location of the aperture in the channel, and on the steady-state current in the upstream beam (which will also determine the incident beam radius, $R_2$). Assume that the steady-state incident beam is matched to the focusing channel, so that $r_1 = R_2 = a_B$. If the beam is apertured at the injection point ($s = 0$), or integer multiples of $\lambda_e$ downstream, the downsteam current perturbation strength will be unchanged. From Eqs. (8) and (25):

$$\eta_3 = \eta_1 . \tag{26}$$

If the beam is apertured at one half wavelength from injection, or integer multiples of $\lambda_e$ thereafter, the downstream current perturbation strength from Eqs. (12) and (25) will be

$$\eta_3 = -\eta_1 . \tag{27}$$

The negative transconductance effect at this location serves to invert the current modulation in the beam. And if the beam is apertured at $s = \lambda_e / 4$, or similar locations, the downstream current perturbation strength from Eqs. (13) and (25) will be

$$\eta_3 = 0 . \tag{28}$$

In this case, the aperture will convert a beam with an arbitrary combination of current and velocity modulation into a beam having pure velocity modulation. The resulting velocity-modulated beam will carry fast and slow space charge waves with current components having equal magnitudes, but opposite signs[2] [12]. This condition can only be approximated by operation of a gridded gun in saturation mode [13].

Note that in Eqs. (26) - (28), there is no dependence on the relative sizes of the aperture and the incident beam. Simply increasing the aperture size changes both the amount of unperturbed current $I_{3,0}$ and the amount of perturbed current $i_3(t)$ passing through the aperture, and therefore has no effect on $\eta_3$, which is essentially the ratio of those currents. Nevertheless, the requirement that $r_2 > r_3$ still holds for Eqs. (27) and (28); if it is violated, the entire beam passes through the aperture, and $\eta_3 = \eta_1$ in all cases.

Finally, note that because the aperture allows the transverse envelope oscillations to affect the nature of the longitudinal space charge waves, these effects can be considered to be a form of transverse-longitudinal coupling. However, this is a fundamentally different type of coupling than the more familiar dependence of the space charge wave speed on the beam radius, which affects beam end erosion [15] as well as fast and slow space charge wave propagation [12]. Experiments at the University of Maryland have confirmed that the latter type of coupling is very weak for small changes

---

[2] We have assumed that the beam extracted from the aperture remains space charge dominated. If the aperture is sufficiently small, this will no longer be the case.

in beam current, and that the coupling from small-amplitude mismatch or breathing-mode oscillations is negligible [16].

4. Conclusions.

We have shown that, with proper placement of an aperture in a uniform focusing channel, the current extracted from the aperture can be made to increase, decrease, or remain unchanged when the current incident on it is increasing. In the context of a gridded electron gun, this is interpreted as a positive, negative, or zero transconductance. This effect can alter the relative strengths of the velocity and current modulation present on an intense beam, which will affect the behavior of space charge waves on the beam emerging from the aperture. Of particular importance is the observation that by placing the aperture at locations where the beam radius is always equal to the Brillouin radius, the current extracted from the aperture will remain constant despite fluctuations in the current extracted from the source. This technique may be of considerable use in the design of accelerators and diagnostics using or measuring intense beams.

This work performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under Contract DE-AC52-07NA27344.

REFERENCES

[1] J.J. Thompson, Philosophical Magazine **44** 293 (1897). Note the apertures shown in figures 1, 2, 4, and 5.

[2] S. Bernal, H. Li, T. Godlove, I. Haber, R.A. Kishek, B. Quinn, M. Reiser, M. Walter, Y. Zou, and P.G. O'Shea, Physics of Plasmas **11** 2907-2915 (2004).

[3] T.P. Hughes, D.P. Prono, W.M. Tuzel, and J.R. Vananne, "Design of Beam Cleanup Zone for DARHT-2," Proceedings of the 2001 Particle Accelerator Conference.

[4] S. Bernal, R.A. Kishek, M. Reiser, and I. Haber, Physical Review Letters **82** 4002-4004 (1999).

[5] S. Bernal, P.G. O'Shea, R. Kishek, and M. Reiser, Proceedings of the 9th Advanced Accelerator Concepts Workshop (2001).

[6] J.R. Pierce, *Theory and Design of Electron Beams*, Section 9.2, Princeton: D. Van Nostrand, 1954.

[7] S.G. Anderson, J.B. Rosenzweig, G.P. LeSage, and J.K. Crane, Physical Review Special Topics - Accelerators and Beams **5** 014201 (2002).

[8] Y. Cui, Y. Zou, A. Valfells, M. Reiser, M. Walter, I. Haber, R.A. Kishek, S. Bernal, and P.G. O'Shea, Review of Scientific Instruments **75** 2736-2745 (2004).

[9] K.R. Spangenberg, *Vacuum Tubes*. New York: McGraw-Hill, 1948.

[10] J.R. Harris and P.G. O'Shea, Journal of Applied Physics, in press (2008).

[11] M. Reiser, *Theory and Design of Charged Particle Beams*, New York: Wiley, 1994.

[12] J.G. Wang, D.X. Wang, and M. Reiser, Physical Review Letters **71** 1836-1839 (1993).

[13] J.R. Harris and P.G. O'Shea, IEEE Transactions on Electron Devices **53** 2824-2829 (2006).

[14] J.R. Harris, J.G. Neumann, K. Tian, and P.G. O'Shea, Physical Review E **76** 026402 (2007).

[15] A. Faltens, E.P. Lee, and S.S. Rosenblum, Journal of Applied Physics **61** 5219-5221 (1987).

[16] J.R. Harris, J.G. Neumann, D. Feldman, R. Feldman, Y. Huo, B. Quinn, M. Reiser, and P.G. O'Shea, "Longitudinal Dynamics in the University of Maryland Electron Ring," in Proceedings of the 2005 Particle Accelerator Conference.

Figure 1. Axisymmetric beam transport system with aperture.

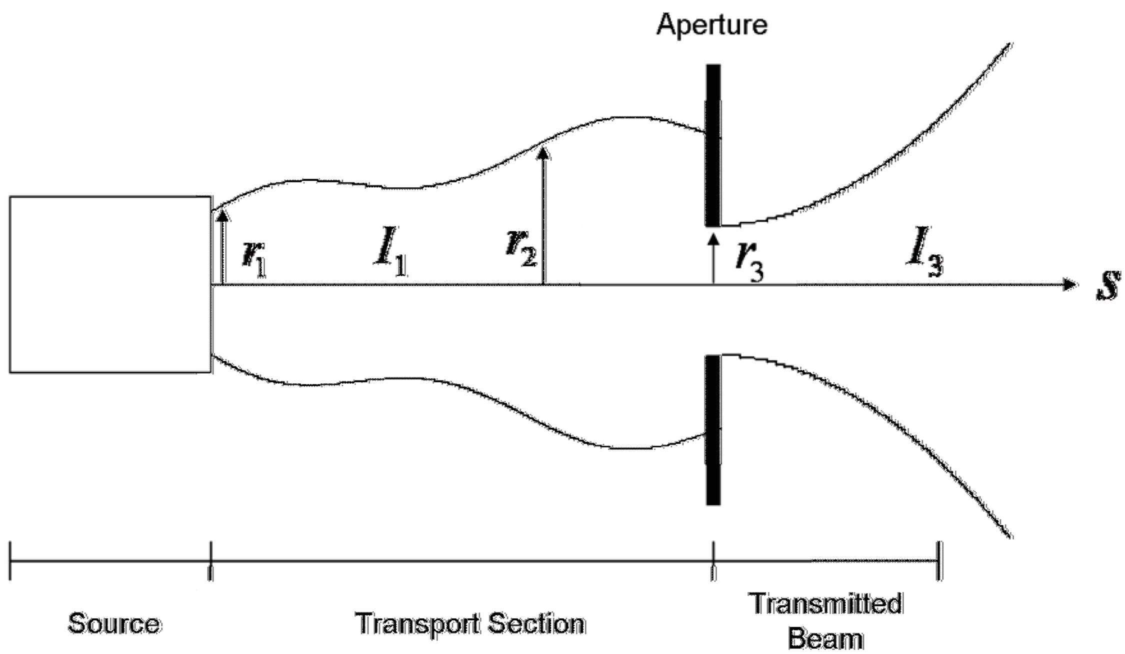

Figure 2. Beam envelope for $I = I_M$ (solid), $I > I_M$ (dot), and $I < I_M$ (dash). $A$ and $B$ refer to the two families of curves in Figure 3.

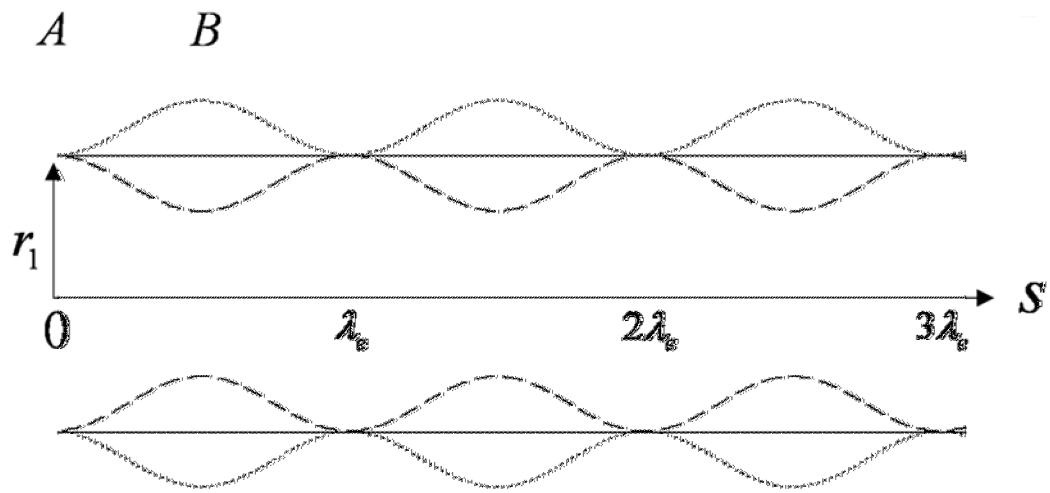

Fig. 3. Relative transconductance ratio ($g_{m,A} / g_m$) plotted as a function of normalized current ($I / I_M$) for several values of $r_3 / r_2$. The curves in group A are for $s = 0$ from Eq. (8) and the curves in group B are for $s = \lambda_e / 2$ as calculated from Eq. (11). The shaded regions are prohibited due to violation of the requirement $r_3 < r_2$. These regions are bounded by the dashed curves, plotted from Eqs. (14) and (15).